

ISC 5935 - Computational Tools for Finite Elements

Homework #6

Assigned 15 October 2014, Due 22 October 2014

http://people.sc.fsu.edu/~jburkardt/classes/fem_2014/homework6.pdf

1. Get a copy of the python program `fem2d_bvp_linear.py` from http://people.sc.fsu.edu/~jburkardt/py_src/fem2d_bvp_linear/fem2d_bvp_linear.html which solves the problem:

$$\begin{aligned} -u_{xx} - u_{yy} &= f(x, y) \\ 0 < x < 1, \quad 0 < y < 1 \\ u(*, 0) &= u(*, 1) = u(0, *) = u(1, *) = 0 \end{aligned}$$

This program uses quadrilateral elements and bilinear basis functions. Modify the program so that the exact solution is

$$u(x, y) = \sin(2\pi x) \cos(\pi y)$$

over the same region, the unit square. You will need to determine the corresponding $f(x, y)$ for this problem, and modify the function `rhs_fn()`. You still want to use Dirichlet boundary conditions, but the boundary value will not be 0 everywhere. You can modify the function `exact_fn()`; then figure out where you need to call that function in order to enforce the boundary conditions.

Turn in: the print-out from the program.

2. Get a copy of the Fenics program `bvp_06.py` from http://people.sc.fsu.edu/~jburkardt/examples/fenics/bvp_06.py which is set up to solve the same problem as `fem2d_bvp_linear.py`. Modify this file in a similar way, so that the exact solution is

$$u(x, y) = \sin(2\pi x) \cos(\pi y)$$

Use the 4x4 element mesh generated by the `UnitSquareMesh()` function, and use piecewise continuous Lagrange elements of order 1 (linears).

Modify the program to print the exact and finite element solutions on a 5x5 mesh of nodes from (0,0), (1/4,0) ..., (1,1).

Turn in: the print-out from the program.

3. Modify your program from problem 2 so that it computes the L2 norm of the error, that is $\|u(x, y) - u^h(x, y)\|$. Then report this error when using:

- 4x4 element grid, piecewise linear basis functions (order = 1);

- 8x8 element grid, piecewise linear basis functions (order = 1);
- 4x4 element grid, piecewise quadratic basis functions (order = 2);
- 4x4 element grid, piecewise cubic basis functions (order = 3).

Turn in: the print-out from the program.
