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Learning How Stars Explode: Supernovae, Turbulent Combustion, and Machine Learning

Brandon Gusto, Tomasz Plewa

Department of Scientific Computing
Florida State University

Contact Information:
Brandon Gusto
blg13@fsu.edu

Introduction

The process of thermonuclear explosion of a white dwarf (WD) star during a Type Ia supernova (SN) event represents an interesting and challenging problem for theorists, computational modelers, and observers. SN Ia are referred to as 'standard candles' because of their predictable brightness, allowing them to be used by astronomers as distance indicators. They are also of interest from the perspective of basic combustion physics, as **the exact mechanism of the explosion remains a topic of active research**. The basic combustion mechanisms believed to occur in SN Ia are relevant to many terrestrial applications as well.

The mode of burning during the explosion process affects the final composition of the SN ejecta, the brightness, and many other properties of interest to observers. It is speculated that the explosion process begins as a **subsonic deflagration** driven by thermonuclear burning near the central region of the star. The deflagration plume is unstable to Rayleigh-Taylor (RT) instability effects as buoyancy forces drive it to the surface of the star. Studies have found that a transition to a **supersonic detonation** is possible within the small-scale turbulence generated by the RT flame [2]. **In this work we examine the exact mechanism that causes the deflagration-to-detonation transition (DDT), and we develop a machine learning model to predict its occurrence.**

Due to the disparate spatial scales involved, a domain decomposition strategy is typically used to simulate large, intermediate, and small scale physics processes independently. The evolution of the spatial scales during the explosion phase of the SN Ia are illustrated in Figure 1. The largest scale is determined by the size of the WD itself. At this scale, the conditions for RT instability are determined. Below this scale the turbulent energy transfers from large to small scales in a cascading fashion. Modelers typically use large-eddy simulation (LES) to study physics from these large scales down to about 1 km of resolution.

As the turbulent flow develops, the scale of the smallest eddies decreases (see the bottom edge of zone 3 in Figure 1). Meanwhile as the density decreases during the explosion, the flame thickness increases. At some point the flame thickness is expected to become comparable to the scale of the smallest eddies, and those eddies are able to penetrate the flame surface before being burned away, fragmenting the flame. **At this point, the distributed flame regime is reached and DDT may be possible**. In this work we use direct numerical simulation (DNS) to study the exact mechanism responsible for the transition.

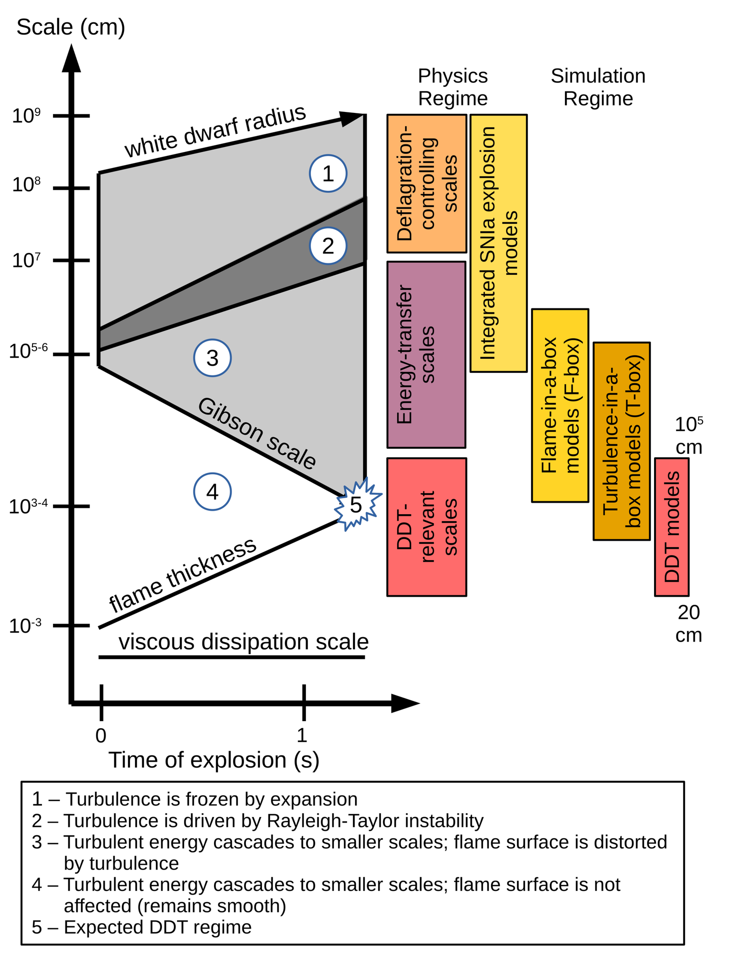

Figure 1: Evolution of spatial scales during the explosion phase of the SN Ia. The physics and simulation regimes corresponding to the spatial scales are highlighted.

Methods

We explore the connection of scales in the exploding WD by developing a model for predicting DDT on small scales based on the Zel'dovich reactivity gradient [3, 1]. **The mechanism involves the coupling of a spontaneously generated reactive wave with the acoustic wave produced by expansion of the gas**. The phase velocity of the reactive wave depends on the spatial gradient of the induction time, τ, which is the estimated amount of time it takes an element to burn its carbon fuel.

A criterion for DDT within a region r_0 of preconditioned fuel, also known as a hotspot, is given by [1] as

$$\sigma_0 < \frac{r_0}{\alpha c_0}, \tag{1}$$

where σ_0 is the standard deviation of induction times in the region, c_0 is the soundspeed, and α is a parameter that depends on other physical quantities such as the density or background velocities. The term r_0/c_0 gives the sound-crossing timescale. A larger value indicates a slower outgoing compressive wave. **A small magnitude of σ_0 with respect to the local induction times indicates that the phase velocity of the reactive wave is high**.

We execute over $25,000$ DNS studies with varying initial hotspot profile shapes, background velocities, and other quantities. **We analyze the initial conditions by computing the reactive versus sound-crossing timescales** according to Equation 1. The results have implications for the possibility of DDT during the explosion phase of SN Ia.

Then we develop a model to predict the potential for hotspots to spontaneously detonate. We introduce a neural-network based model that solves a binary classification problem. The network is trained on the DNS database. Several features selection strategies are used. A naive strategy is used that considers the spatial profile of induction time or other variables. We also explore another strategy, the Khokhlov strategy, that considers the timescales in Equation 1 among other quantities.

The network training process is illustrated for the simplest naive strategy in Figure 4,

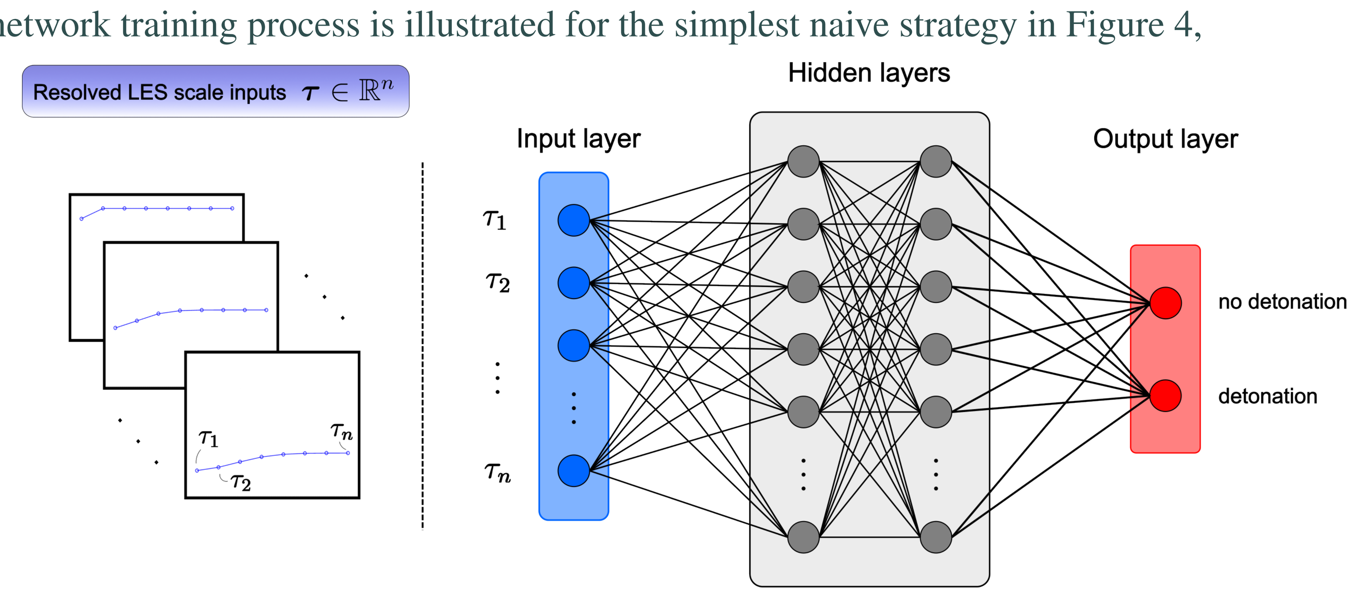

Figure 2: Overview of the neural network based learning strategy. Hotspot profiles are passed as inputs to the network, with labels of 'no detonation' and 'detonation' being learned. Several hidden layers are used which may be convolutional layers, fully connected layers, or a combination thereof.

with induction time profiles being passed as inputs, and labels of 'no detonation' and 'detonation' being learned. We use a convolutional neural network (CNN) for the naive approach and an artificial neural network (ANN) for the Khokhlov approach.

Results

We show the results of the DNS of a single hotspot configuration in Figure 3.

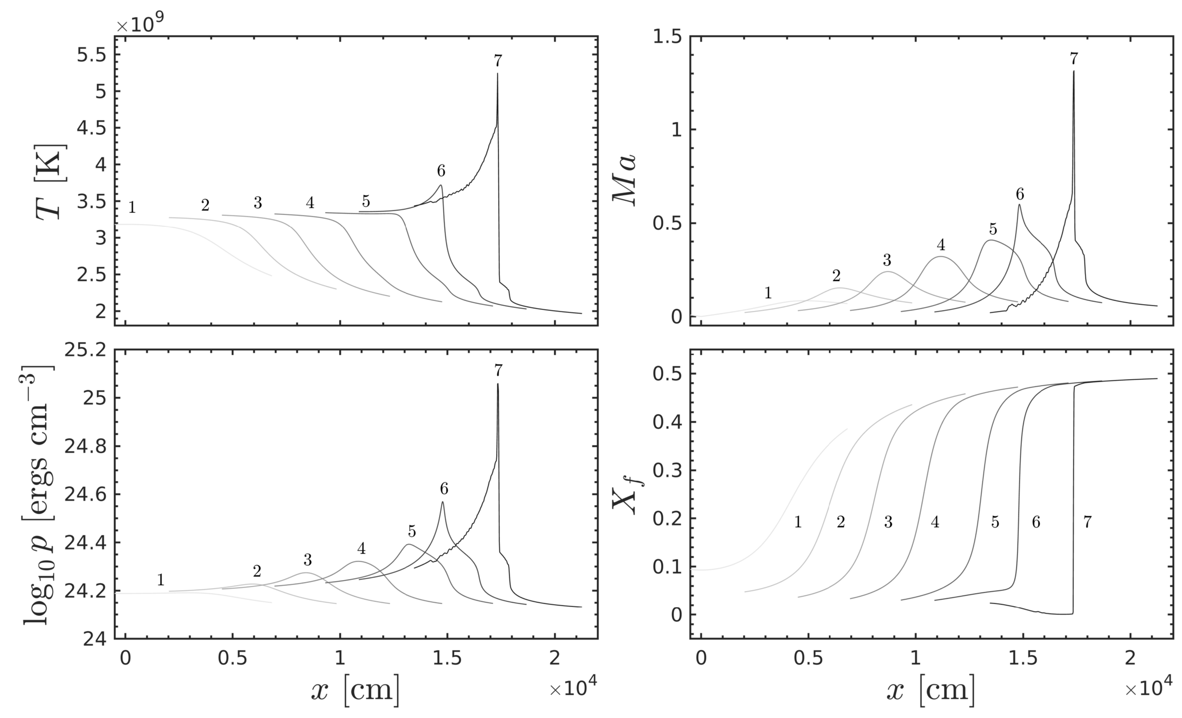

Figure 3: Direct numerical simulation of a hotspot. Shown is the evolution of temperature (top left panel), Mach number (top right panel), pressure (bottom left panel), and carbon abundance (bottom right panel) at seven times during the evolution.

The evolution in temperature, Mach number, pressure, and carbon abundance is plotted at seven times. Initially at time t_1, burning in the center of the region ($x = 0$ cm) causes expansion, a buildup of pressure, and the eventual creation of an outgoing compressive wave by time t_2. As the compressive wave travels outward, the reactive wave behind it continues to generate overpressure. The compressive wave steepens into a shock as it collides with slower, denser material ahead of it, eventually creating a shock-reaction structure (a detonation wave).

Analysis of the whole DNS dataset using Equation 1 on the reactive versus sound-crossing timescale plane is shown in Figure 4.

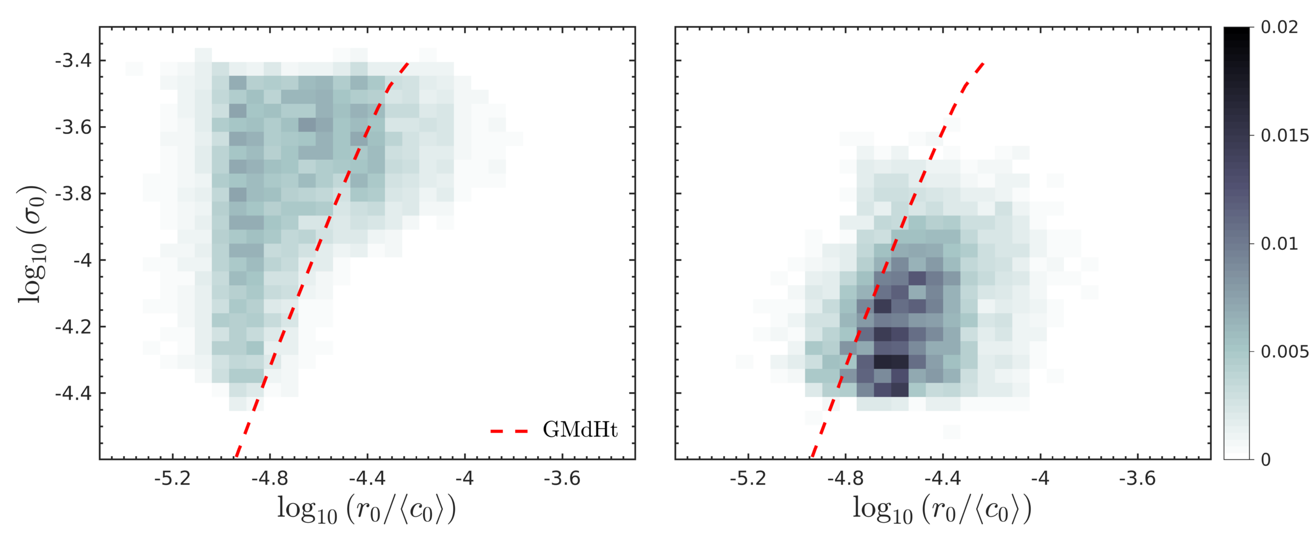

Figure 4: Results of the DNS dataset on the reactive versus sound-crossing timescale plane, divided into non-detonating (left panel) and detonating (right panel) subsets.

In the left panel, the non-detonating samples are concentrated in the upper left portion of the plane, indicating that **large reactive timescales and small sound-crossing timescales are unfavorable for detonation formation**. Likewise the detonating samples are concentrated lower and more to the right, indicating that **smaller reactive timescales and larger sound-crossing timescales are more favorable**.

Different neural network strategies are trained using the DNS dataset. All of the strategies demonstrate good accuracy on both the training data and the validation data. A $90/10$ split of the whole DNS dataset is used to form the training and validation sets, respectively.

Finally we present the results of the trained network on predicting the onset of detonation from hotspots in reactive LES. In Figure 5 we show the network's confidence in a detonation occurring (indicated by the colormap), with 0 being the lowest confidence and 1 being the highest, on the δ_t versus δ_x plane, where δ_t is the time before the detonation formation and δ_x is the absolute distance from the location of the hotspot from which the detonation was formed. We find the network has a good ability to positively identify imminent detonations roughly 2×10^{-4} to 4×10^{-4} seconds prior to their formation.

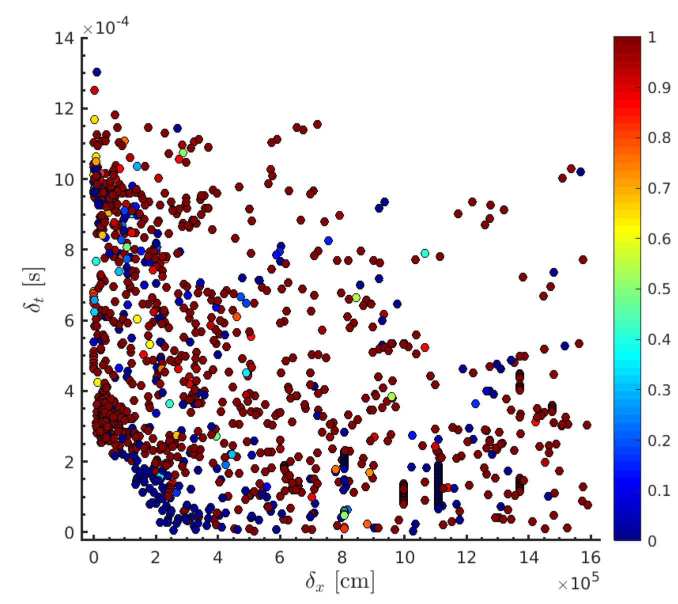

Figure 5: Performance of the network trained using the Khokhlov feature selection strategy when used to detect hotspots in LES of reactive turbulence.

References

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