

# NUMERICAL OPTIMIZATION

## SYLLABUS

- Introduction
- The NCP problem
- Characterization of single variable unconstrained minimum
- Necessary and sufficient first and second order conditions for an unconstrained minimum.
- Examples
- **Unconstrained minimization methods**
- Properties of a quadratic function
- Convex and concave functions
- Method of bisection
- Newton's method
- The secant method
- Regula -falsi method
- Univariate minimization
- Unimodal functions
- Fibonacci search for univariate minimum and algorithm
- Golden section method and algorithm
- Quadratic fit-polynomial interpolation for univariate minimum
- Cubic fit polynomial interpolation
- Methods for minimization of multivariate smooth functions
- Direction of descent
- Global convergence of descent methods
- Computing step-length along direction of descent
- Armijo rule for step length

- Goldstein Armijo test for step length
- The Wolfe test for step length
- Step length for a quadratic function
- Computation of good search directions
- Method of steepest descent
- Examples
- Newton's method for multivariate functions
- Problems with Newton's method
- Modification of Newton's method for indefinite Hessian
- Methods based on Choleski decomposition
- **Quasi-Newton Methods**
- Matrix outer products and rank one matrices
- Symmetry and positive-definiteness in Hessian approximations with Q-N method.
- Algorithmic implementation of Quasi-Newton methods
- Davidon Fletcher Powell (DFP) Quasi-Newton (Q-N) Algorithm
- Examples
- Positive definiteness of DFP Q-N update to the Hessian.
- Finite-step convergence of DFP Q-N method
- Davidon Fletcher Powell Q-N method with cubic interpolation for the step-length.
- Q-N method for positive definite quadratic form
- Variable metric form of the Quasi-Newton descent methods.
- Broyden-Fletcher-Goldfarb-Shanno (BFGS) Q-N method
- Use of Sherman Morrison Inverse matrix formula
- Variational metric properties of DFP and BFGS Q-N Methods.
- Scaling issues related to Quasi-Newton method
- Eigenvalues of Hessian and the interlocking eigenvalues Theorem.
- Choice of appropriate scale factors and self scaling Q-N algorithm.
- Limited Memory Quasi-Newton Methods
- **Conjugate Gradient algorithms**
- Conjugate directions Theorem
- Expanding Subspace Theorem
- Geometric 2-D interpretation of conjugacy concept and polarity.
- Conjugate gradient (C-G) algorithm and examples.
- C-G Theorem

- Fletcher -Reeves C-G algorithm for nonlinear functions.
- C-G methods viewed as an optimal process.
- Convergence rate of the C-G method
- Hessian with clustered eigenvalues and preconditioning
- Examples
  - Different C-G algorithms with various line-search methods
  - Polak-Ribiere C-G algorithm
  - Restart methods for nonlinear C-G algorithm
  - Powell's (1977) restart C-G method using Beale's condition
  - Memoryless Q-N like C-G methods: Codes and algorithms.
  - Scaling and preconditioning in C-G methods
  - Variable-storage Q-N like C-G Methods : Codes, Algorithms and Examples
- Truncated Newton methods and examples
- Use of scientific minimization software libraries. ( NAG, IMSL, NEOS)
- **Constrained Optimization**
  - First and second order conditions : theory
  - Karush- Kuhn Tucker (KKT) Theorem
  - Linear and nonlinear constraints and the active set approach
  - Penalty and barrier functions methods
  - Reduced gradient and gradient projection methods
  - Exact penalty methods
  - Augmented Lagrangian Methods
  - Sequential Quadratic Programming ( SQP)
  - Reduced Gradient methods (GRG)
  - Rate of convergence of constrained minimization methods
  - Nondifferentiable constrained minimization methods
  - The subgradient bundle algorithm for nondifferentiable minimization
- **Global Minimization methods**
  - Simulated annealing methods theory and applications
  - Genetic algorithms-Theory and applications
  - Other global search methods
  - Experience with simulated annealing and genetic algorithms