Exploring a Multi-Resolution Approach within the Shallow-Water System

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Abstract The goal of this project is to explore how multi-resolution grids effect global shallow-water models. Our approach is to generate high-quality Spherical Centroidal Voronoi Tessellations (SCVTs) that result in two dominate resolutions; where one resolution (the fine scale) covers the region of interest and the other resolution (the coarse scale) covers the rest of the sphere. The two dominate scales are separated by a mesh transition zone that, like the fine and coarse scales, is easily altered through the user-specified mesh-density function. We apply our finite-volume solver along with these variable resolution SCVTs to the standard suite of shallow-water test cases in order to highlight the strengths and weakness of this multi-resolution approach.

Goal of Research

We are attempting to create variable resolution grids that perform comparably to uniform resolution grids. These new grids could allow us to obtain a "better" solution in a region of the grid, and are well suited for regional modeling. One thing to note, is that we are not looking for grids that produce a better solution, as we don't expect this to happen; we are simply looking for grids that don't perform significantly worse than an uniform grid. Later we could attempt to tackle the grids to specific parameterizations of other physical processes, to obtain a better result than the parameterized version of the uniform grid, especially in a specific region. We will be using spherical centroidal Voronoi tessellations for our grids.

Mathematical Model

This research is using a newly developed finite volume solver, and the shallow water equations for the testing purposes of our grids. The shallow water equations are given as follows:

Momentum Equations:

\[ u_t + uu_x + vv_y - f v = -gh_x \]
\[ v_t + vv_x + uu_y + f u = -gh_y \]

Continuity Equation:

\[ h_t + uu_h + vv_h = 0 \]

Density function for generators

To create our variable resolution grids, we had to come up with a density function that would be used to generate our SCVT. The criteria we were looking for was:

- Radially Symmetric
- Smooth Transition Region
- Maximum Value of 1

The density function that we came up with is as follows:

\[ \rho = \left(1 + \left(\frac{r}{r_m} - 1\right)^2\right) \left(1 - \frac{r}{r_m}\right) \]

In this equation, \( r_m \) is the center of the transition region (in radians) from the center of the density function. \( r \) is the distance in radians from the center of the density function, \( \rho \) is a parameter to control the width of the transition region, and \( m \) is the parameter of density this function should output. Using the parameters of \( r_m = 0.15, m = 0.0001 \) gives us a density function that can be seen below.

Grids for Simulation

For our test case, we generated SCVTs with four different numbers of generators. The number of generators used were 2562, 10242, 40962, and 163842. These correspond to grid spacings of about 480km, 240km, 120km, 60km, respectively. For the variable resolution grids, we wanted a range of grids we could test. By changing \( m \), we could control the grid spacing difference between the coarse region and the fine region by some factor. We generated 5 different grids at each of the grid point, using \( m \) to give us a factor of 1, 2, 4, 8, and 16 between the coarse and the fine regions. When the factor is 16 it means that the fine region cells are about 16 times smaller than the cells in the coarse region. The different factors produce grids that look like the following:

References