# Supervised Aggregation of Classifiers Using Artificial Prediction Markets

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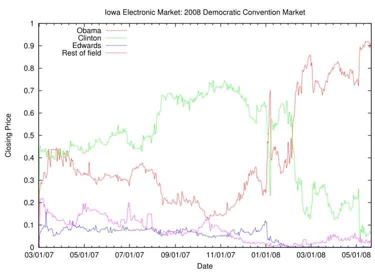
## Prediction Markets

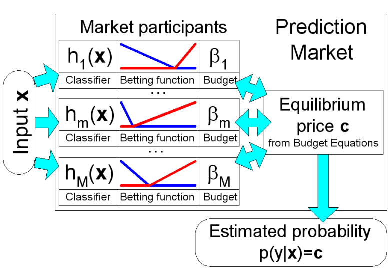
- Forum where contracts are traded on future outcomes.
- Contracts pay contingent on the outcome.
- Trading price of contracts reflects combined knowledge and experience of participants.
- Trading price is an estimator of the probability.
- Can predict outcomes of elections, sporting events, and foreign affairs.
- Were demonstrated to be more accurate than polling or individual experts.




Trading prices of contracts on democratic nominees for the 2008 presidential election.

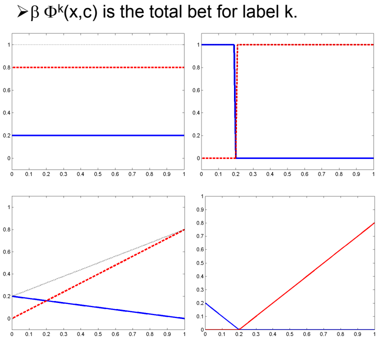
## Idea

- Reinterpret events as *instances*, future outcomes as instance *labels*, and participants as *classifiers*.
- For each instance, classifiers "purchase" contracts for each possible label.
- The trading price for each label determines the classification.




## Betting Functions

- Classifiers are functions of instance data.
- Purchasing contracts is a function of both instance data and price.
- Introduce *Betting Functions* that allot budget for contracts based on *classifier* and *price*.
- $\beta\,\Phi^k(x,c)$ is the total bet for label k.



Four betting functions: Constant, Aggressive, Linear, and Piecewise Linear from left to right and top to bottom respectively.

- Bet "rationally".
- If a contract is priced 1 then potential winnings. amount to 0 while potential losses amount to 1. So $\Phi(x,1) = 0$.
- If a contract is priced 0 then potential winnings. are infinite with no potential loss. So $\Phi(x,0)$ ¸ 1.
- Increased price should correspond to decrease in bet and so $\Phi(x,c)$ is monotonically decreasing in c.

## Conservation of Budget

- Budget sum should be constant.
- Accuracy depends on the budget configuration.
- If budgets vanish or "blow up" then the prediction is not useful.
- Total potential winnings and bets should equal for every label.
- Defines the equilibrium price (unique).

$$\text{Total bet} = \text{Total winnings}$$

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\beta_m\phi_m^k(\mathbf{x},\mathbf{c}) = \sum_{m=1}^{M}\beta_m\frac{\phi_m^y(\mathbf{x},\mathbf{c})}{c_y}$$

**Theorem**

**Budget Conservation.** *The total budget $\sum_{m=1}^M \beta_m$ is conserved after the* **Market Update**$(\mathbf{x}, y)$*, independent of the outcome $y$, if and only if there exists $n \in \mathbb{R}_+$ such that*

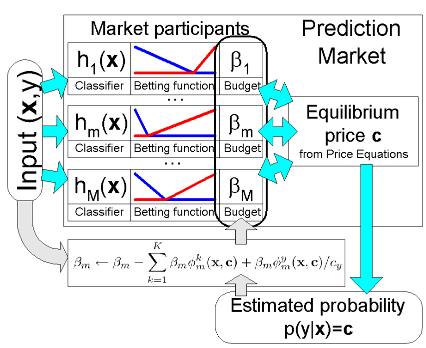
$$\frac{1}{\mathbf{c}_k}\sum_{m=1}^{M}\beta_m\phi_m^k(\mathbf{x},\mathbf{c}) = n, \quad \forall k = 1,\ldots,K \qquad (1)$$

**Theorem**

**Monotonic Betting Functions.** *If all betting functions $\phi_m^k(\mathbf{x}, c_k)$, $m = 1,\ldots,M$, $k = 1,\ldots,K$ are continuous and monotonically decreasing, then for each* **Market Update**$(\mathbf{x}, y)$ *there is a unique price $\mathbf{c} = (c_1,\ldots,c_K)$ such that the total budget $\sum_{m=1}^M \beta_m$ is conserved.*

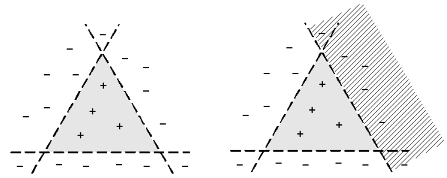
## Learning

- Budgets describe the prediction accuracy of each participant.
- The goal is to learn the budget configuration that improves the market's prediction accuracy.
- Each participant bids for contracts and are rewarded based on correct prediction.

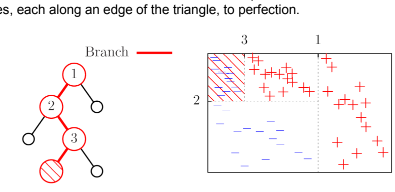



## Specialization

- Classifiers that describe a subdomain of the instance space are called *specialized classifiers*.
- Specialized participants only bet if an instance is in the classifier's domain of specialization.



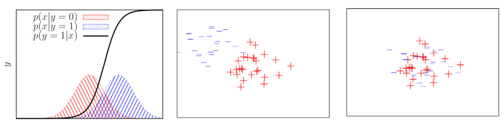
The prediction market can boost six classifiers specialized on six half planes, each along an edge of the triangle, to perfection.



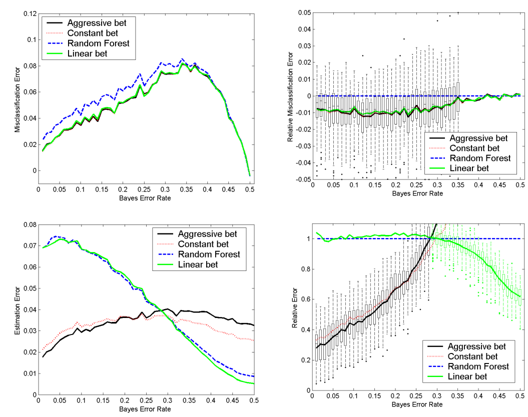

Decision tree branches are another example of a specialized classifier. Each vertex along the branch corresponds to the cut in the instance space. Decision tree branches are perfect classifiers of training data in their subdomain.

## Results

- Results on both synthetic and real data sets.
- Synthetic data sets are 100D Gaussian mixtures with varying Bayes errors.
- Real data sets are from UCI repository. There are 24 total.



Examples of 1D and 2D Gaussian mixtures. The left demonstrates Bayes error, the region of overlap. The middle has Bayes error 0 (easy) and the right has Bayes error 0.5 (hard).




The prediction market and random forest were trained on 100 samples of 100D Gaussian mixtures each of size 200 over 50 increments of Bayes error. The probability error was computed with $E\|p - p^*\|_2$ over a sample size of 10000.

| Data | Size | RF | CB | LB | AB |
|---|---|---|---|---|---|
| breast-cancer | 699 | 2.4 | **2.3** | **2.3** | **2.3** |
| car | 1728 | 2.2 | **1.1** | 1.3 | **1.1** |
| sonar | 208 | 14.5 | **13.1** | 13.4 | **13.1** |
| vowel | 528 | 7.5 | **6.9** | 7.0 | **6.9** |
| credit-screening | 690 | 11.9 | 11.8 | **11.7** | **11.7** |
| cylinder-bands | 541 | 18.9 | **18.7** | **18.7** | **18.7** |
| ecoli | 336 | 12.6 | **12.5** | 12.6 | **12.5** |
| german | 1000 | **22.9** | **22.9** | **22.9** | **22.9** |
| glass | 214 | 2.1 | **1.8** | **1.8** | **1.8** |
| image | 210 | 6.6 | 6.5 | 6.5 | **6.4** |
| ionosphere | 351 | **6.6** | **6.6** | **6.6** | **6.6** |
| letter-recognition | 20000 | **3.2** | **3.2** | **3.2** | **3.2** |
| poker | 25010 | 38.0 | **35.7** | 35.9 | **35.7** |
| satimage | 4435 | 8.7 | **8.6** | **8.6** | **8.6** |
| yeast | 1484 | 35.4 | 35.3 | **35.2** | 35.3 |
| balance-scale | 625 | 16.5 | 16.8 | 16.7 | **16.1** |
| connect-4 | 67557 | 19.7 | **16.6** | 16.8 | **16.6** |
| isolet | 1559 | 7.5 | **7.4** | **7.4** | **7.4** |
| kr-vs-k | 28056 | 21.4 | 11.0 | 11.7 | **10.9** |
| kr-vs-kp | 3196 | .9 | **.4** | .5 | **.4** |
| madelon | 2000 | 26.3 | **20.4** | 20.8 | 20.5 |
| magic | 19020 | 11.8 | **11.6** | **11.6** | **11.6** |
| musk | 6598 | **0** | **0** | **0** | **0** |
| gene-sequences | 3190 | 4.6 | **4.4** | 4.5 | **4.4** |

The prediction market and random forest were trained and tested on 100 random samples with 90% of each data set used for training and 10% used for testing. Image (2100), satimage (2000), and poker (10^6) provide test sets. The table provides the misclassification rates for Random Forest (RF), Constant Betting (CB), Linear Betting (LB), and Aggressive Betting (AB).

## References

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