Simulation of Small-Scale Magnetic Effects on Unstable Interfaces using Tracking Methods

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Introduction

Type Ia supernovae are spectacular explosions in space thought to be the final episode in the lives of accreting massive degenerate white dwarf stars. A white dwarf is the end product of stellar evolution in which original matter has been fused to carbon and heavier elements. Provided the white dwarf is a member of a binary system and accretes matter from a companion star, it slowly contracts due to the force of its own gravity. In the process of temperature increase in the core, a subsonic reactive flame front (deflagration) is born. The flame propagates through the stellar interior via thermal electron conduction. The thermonuclear burning is believed to eventually transit from the deflagration to a detonation, which consumes the whole star. The theoretical foundation for that transition process does not exist.

The luminosity of the explosion rivals that of an entire galaxy. It is believed that all Type Ia supernovae explode starting from similar initial conditions and thus produce similar intrinsic luminosities. This justifies using Type Ia supernovae as "standard candles". This property is extremely useful because given the apparent magnitude (the "brightness" as seen by an observer) of the explosion, knowledge of the actual luminosity allows for calculation of the distance to that object. Such information has applications in cosmology and allows for probing the early physics and geometry of the universe.

The supernova flame is subject to numerous instabilities, most notably Rayleigh-Taylor (RT) and Landau-Darrieus (LD) instabilities. The RT instability is a buoyancy-related phenomenon due to light fluid (ash) being accelerated into heavy fluid (fuel). This causes well known finger-like structures and is the source of large scale turbulence.

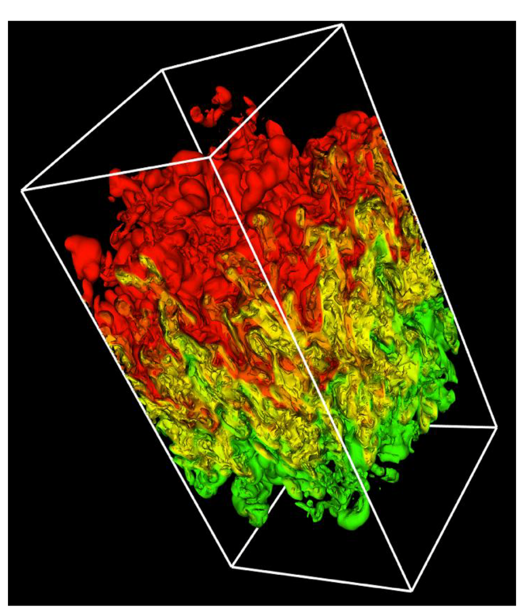

Figure 1 : A rendering of density from a validation simulation of the Rayleigh-Taylor instability. [1]

The LD instability is geometric in nature and occurs on a scale that is 10^8 times smaller than RT. Here the density jump across the flame causes refraction of streamlines as the flow crosses the flame and enters the hot ash. The theory behind LD predicts a stabilization mechanism – a series of cusps – which inhibits growth of the instability.

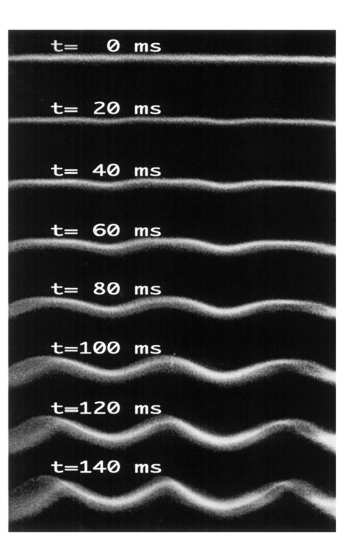

Figure 2 : Images taken from high speed film of growth of the LD instability. [2]

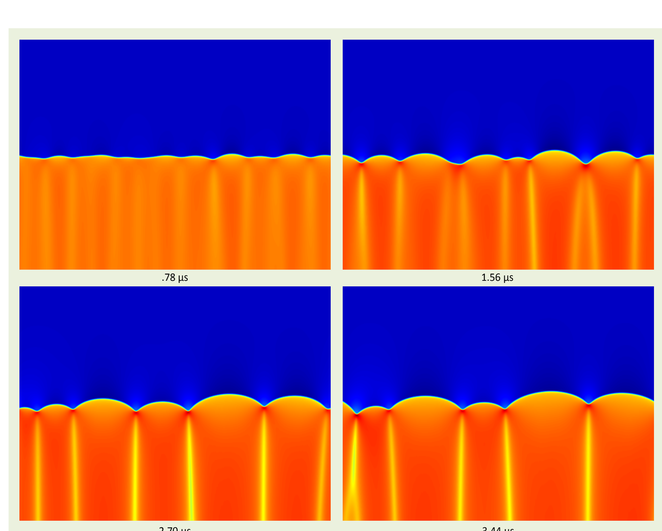

Figure 3 : Selected snapshots of a carbon nuclear flame simulation which shows LD cusps coalescing with time. [3]

Direct numerical simulations of Type Ia supernovae are computationally unfeasible due to the extreme range of scales involved, as depicted in the diagrams below. [3], [4]

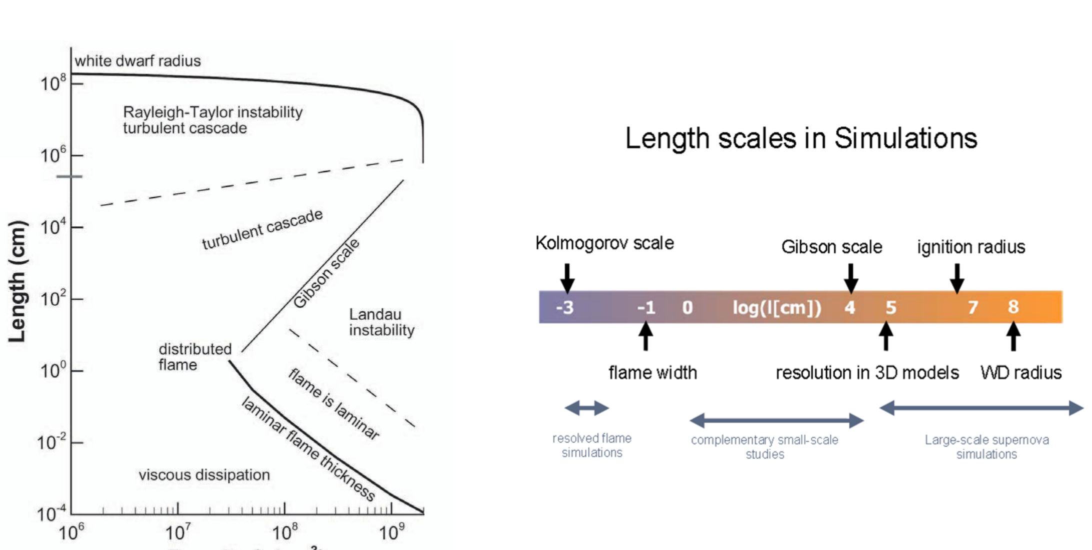

Figure 4: Illustrations of the length scales involved in Type Ia supernova simulations

To make the problem computationally tractable, a Large Eddy Simulation approach is used with a subgrid-scale flame model. The flame front evolution might be described either by solving the appropriate advection-diffusion-reaction equations or with the help of one of the existing front tracking methods supplemented by the flame physics. In the latter case, a level set approach (G-equation) is the most popular. Another possibility is the volume of fluid (VOF) method. We are currently reviewing the suitability of both approaches (in collaboration with M. Sussman).

Landau-Darrieus Instability

Flame curvature plays an important role in the supernova physics because it determines the effective burning speed of the flame. This is due to the fact that the flame is propagating into the fuel in one-dimensional fashion in the direction perpendicular to the flame surface. This local flame speed is called the laminar flame speed and depends only on the physical conditions in the fuel. Thus the total amount of burnt fuel depends not only on flame properties but also its geometry. Therefore any mechanism that introduces "wrinkles" in the flame should be either captured (resolved) or modeled.

Consider a planar, infinitely thin propagating surface which separates a region of hot ash from cold fuel. This surface, which is the flame, releases energy via exothermic nuclear reactions. The temperature and density are discontinuous across the flame. Based on conservation laws across the flame surface, the tangential component of the fluid velocity is continuous while the normal component experiences a jump. Therefore streamlines will bend towards the flame normal. These perturbations die out at long distances from the front, and the result is diverging and converging flow at different parts along the flame. Assuming the flow is incompressible, diverging flow implies decreasing flow speed and increased forward propagation speed of that section of the flame. The opposite happens for converging flow; that part of the flame is decelerated. Thus we see that any initial flame front perturbation will grow with time. This is the physical mechanism behind the LD instability.

In the general case with gravity included, the LD growth rate is given by:

$$\omega = \left(\frac{1+q_c}{2+q_c}\right) \times \left\{-\hat{k} + \sqrt{\left[1+\left(\frac{q_c(2+q_c)}{(1+q_c)}\right)\right]\hat{k}^2 - 2(2+q_c)(1-Ma^0)\hat{l}_T\hat{k}^3 - \frac{q_c(2+q_c)}{(1+q_c)}g\hat{k}}\right\}$$

In the limit of no gravity, the classical result is obtained:

$$\omega = \left(\frac{1+q_c}{2+q_c}\right) \times \left\{-1 + \sqrt{1+\left(\frac{q_c(2+q_c)}{(1+q_c)}\right)}\right\}\hat{k}$$

This gives $\omega > 0$ for all $q_c > 0$, which implies that the flame is unstable to perturbations at all wavelengths.

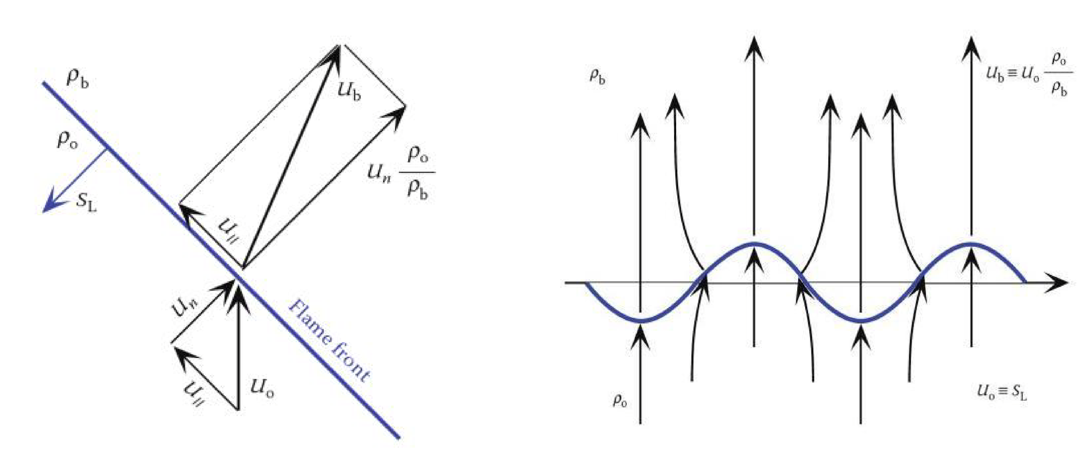

Figure 5 : (left panel) The split of the flow velocity into tangential and normal components is shown. Notice that the tangential component remains fixed, but the normal component jumps. (right panel) The LD mechanism in the context of a wrinkled flame. [6]

Biermann Battery

In this project we aim to study the effects of small-scale magnetic fields on the flame evolution. Such fields are created when gradients of electron temperature and electron number density are misaligned, which is expected at the convoluted material interface. In our approach, a front tracking method will allow accurate calculation of these gradients.

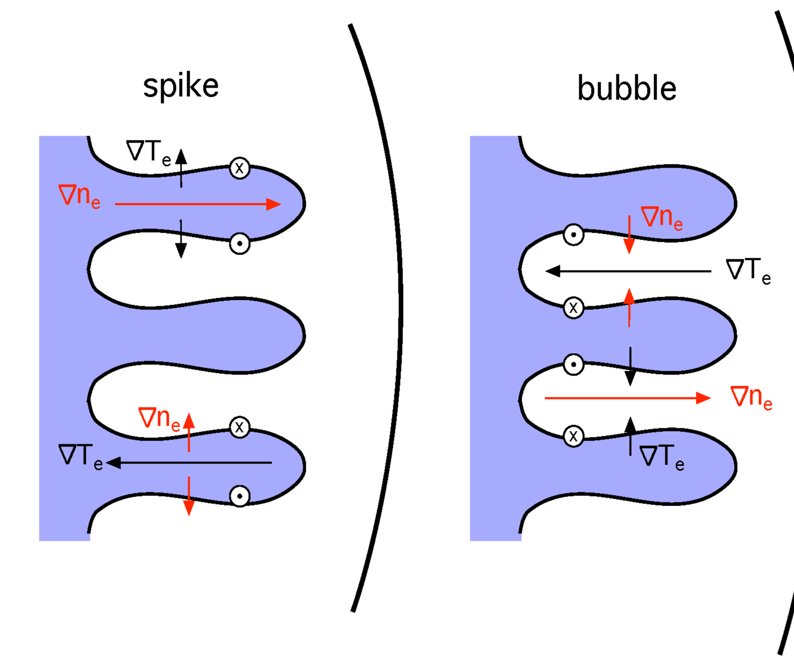

Figure 6 : The Biermann battery mechanism at the RT-unstable interface. The magnetic field is generated at the interface due to misalignment of gradients of electron number density and electron temperature. A similar situation arises at the wrinkled flame front [7]

Computational Model

We use the FLASH code to numerically solve the compressible Euler equations in conservation form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathrm{v}) = 0$$

$$\frac{\partial \rho \mathrm{v}}{\partial t} + \nabla \cdot (\rho \mathrm{v}\mathrm{v}) + \nabla P = \rho \mathrm{g}$$

$$\frac{\partial \rho \mathrm{E}}{\partial t} + \nabla \cdot [(\rho \mathrm{E} + P)\mathrm{v}] = \rho \mathrm{v} \cdot \mathrm{g}$$

For reactive flows, a separate advection equation is solved for each species:

$$\frac{\partial \rho \mathrm{X}_\ell}{\partial t} + \nabla \cdot (\rho \mathrm{X}_\ell \mathrm{v}) = 0$$

The code uses the Piecewise Parabolic Method (PPM) which is a Godunov-type scheme using high-order reconstruction. Adaptive mesh refinement is provided by the PARAMESH package.

Front tracking will be used to locate the flame and obtain accurate gradient information, required to calculate the Biermann battery source term. One possible approach involves evolving the level set whose zero is identified as the flame front.

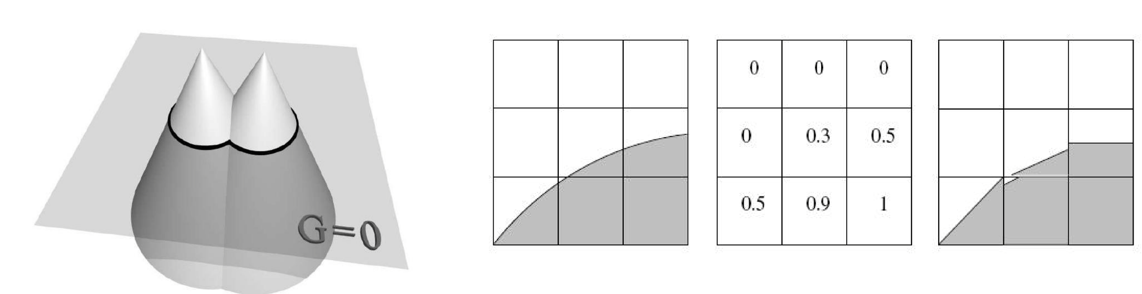

Figure 7 : (left panel) Illustration of a signed distance function representing the interface being tracked using the level set method. [8] (right panel) Illustration of volume fractions and interface reconstruction using the volume of fluid method. [9]

Future Work

The LD and RT instabilities are often discussed as two distinct phenomena. However, LD can be obtained in the limit of RT with negligible gravity. In other words, RT is a generalized version of LD.

We will study the effects of intrinsically generated magnetic fields via the Biermann battery mechanism in the context of both LD- and RT-unstable interfaces. It has been shown that large scale magnetic fields can quench flames, thus it is conceivable that small-scale fields may affect the flame evolution. We will implement an interface tracking method, either a level set or volume of fluid algorithm, which will provide accurate information required to calculate the magnetic field source term. Our ultimate goal is to use this approach to study thermonuclear deflagrations in Type Ia supernova explosions.

References

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