Feasibility of Core Collapse SN Experiments

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SUPERNOVA SETTING For much of a core collapse SN's development its internal processes are hidden from view. In particular, neither the hypothesized Standing Accretion Shock Instability (SASI, [1]), believed to be the key agent in driving the explosion, nor the related Advective Acoustic Cycle (AAC, [2]) have been experimentally verified. The current theory on SASI is that the region between the shock front and the surface of the proto-neutron star (PNS) may be decomposed into two sections at a distance called the gain radius. Below the gain radius neutrino cooling dominates while heating increases the internal energy of the matter at higher radii. Inside the cooling region neutrinos extract energy from the accreted material allowing it to settle to the surface of the PNS. In contrast, the neutrino-heated region acts like a boiler plate for the incoming shocked envelope material. As the result, the region between the gain radius and the shock becomes convectively unstable. The emerging convective pattern contains downdrafts that penetrate deep into the heating region, where the heating turns them into buoyantly rising plumes as can be seen in the background image (plumes are red, downdrafts are blue, and the "boiler plate" is shown in yellow).

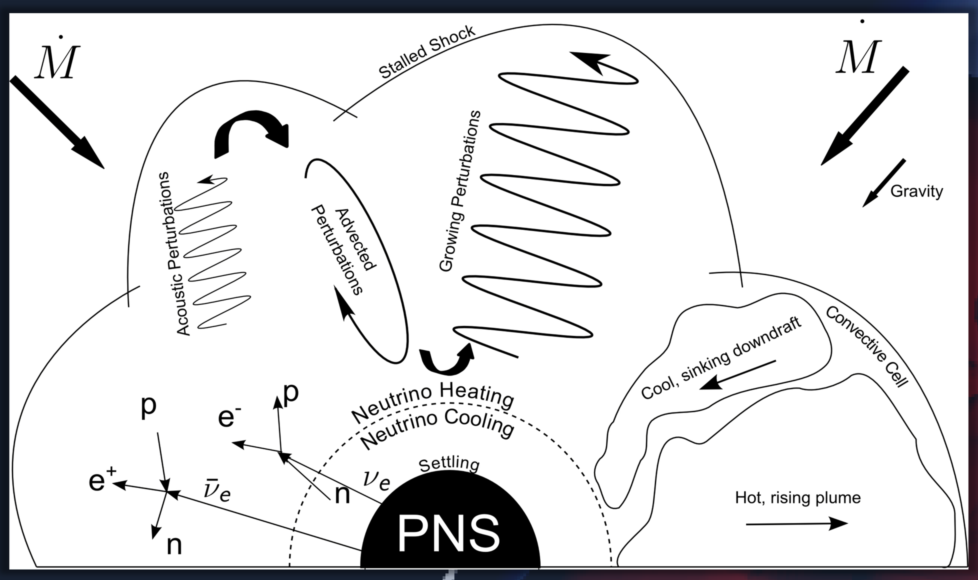

Fig. 1: Illustration of the various mechanisms behind a SASI and the AAC.

Ryutov et al. [3] have conducted a dimensional analysis of the SN problem and demonstrated that the relevant conditions can be obtained in the laboratory high-energy-density experiments. He has shown that that experiments and astrophysical systems behave similarly if certain dimensionless quantities are likewise comparable. Of particular interest to this work is the Euler number, $Eu = v(\rho/P)^{1/2}$, which has similar meaning as the Mach number. Our supernova explosion models indicate $Eu \approx 0.6$ in the convectively unstable post-shock region.

EXPERIMENTAL SETTING Recently, Ohnishi et al. [4] have proposed a design for laser-driven SN-motivated SASI experiment. Their design utilizes a spherical shock reflected off of a small, dense hemisphere. In the successful design, the emerging post-shock region should produce conditions required for the Advective Acoustic Cycle (AAC) to develop.

We note that the above scenario does not include certain physics effects present in the supernova setting such as heating and cooling, and gravitational acceleration. It is natural to ask whether those missing components are necessary for capturing supernova explosion dynamics.

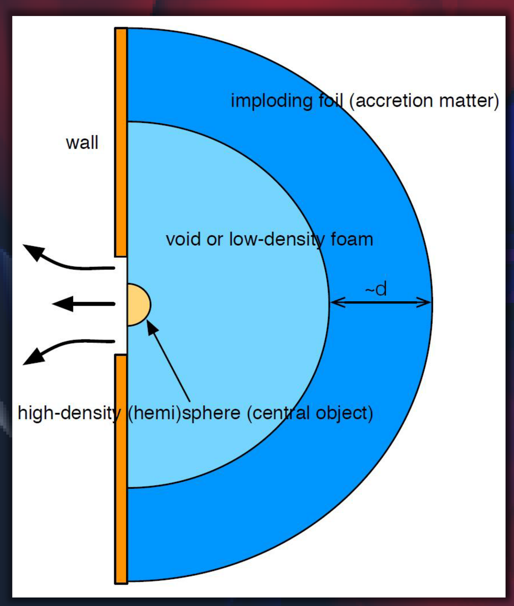

Fig. 2: Experimental design proposed by Ohnishi et al. [4].

EXPERIMENTAL PREREQUISITES The primary effect of the neutrino cooling is the extraction of energy from the shocked material. This allows for the matter to be accreted onto the PNS surface and effectively decouple from the flow, at least as far as convective processes are concerned. Therefore, this part of the flow does not need to be represented in the experiment. This motivates the presence of small openings surrounding the reflector in the Ohnishi et al. design.

In supernovae, the gravity also participates in the extraction of energy from the post-shock region. This results in deceleration of the flow with radius. In contrast, in an experimental setting, the flow accelerates as it converges toward the center (nozzle-like flow), decreasing the advective time of shocked gas elements and thus limiting the time available for instabilities to develop. Following Marek & Janka [5], the supernova shock is stalled for 200-800 ms, and the post-shock advection time is $\tau_{adv} \approx 30$-50 ms. Comparing the two timescales, one can see that the time required for AAC is $\tau_{AAC} \approx 10\tau_{adv}$. This is comparable to the explosion timescale, and in the experimental setting corresponds to the amount of time the shock has to remain quasi-stationary. This is the primary prerequisite for the successful experimental SASI design. It also implies that the experimental system must be stable to radial perturbations for at least τ_{AAC}.

The effect of the increased advection rate at small radii may not be important if the post-shock flow has enough time to develop convective behavior. However, convectively unstable situation may be the most difficult to produce in the first place. In the absence of convective instability, one has to work with a shocked gas characterized by modest radial gradients and smoothly advected through the inner boundary. Then the goal is to study stability of this flow subject to perturbation induced in the upstream region. One can also envision a situation in which a heat source is created inside shocked gas, as we discuss below.

NUMERICAL MODEL We study both the supernova and experimental settings by means of a high-resolution hydrocode combining adaptive mesh discretization with a PPM hydro kernel. We solve the compressible Euler equations with the ideal gas equation of state in spherical coordinates, with the following boundary conditions.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$
$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \otimes \vec{u} + p) = \rho \vec{g}$$
$$\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho \vec{u} E + \vec{u} p) = \rho \vec{u} \vec{g}$$

Eqn. 1: Euler equations with gravity source terms

At the outer boundary, we impose a supersonic inflow. The inner boundary condition for supernova systems is that of the "leaky boundary" of Blondin et al. [1] with the zero gradient of density and hydrostatic pressure profile mimicking conditions at the PNS surface. Blondin et al. solve for the steady state velocity analytically and fix the interior velocity to the analytic value, which effectively controls the height of the shock. These conditions allow for a variable mass flow rate through the inner boundary. The gravitational source term is normalized in such a way that $GM_{pm}=0.5$, where M_{pm} is the central point mass, as done in Blondin et al.

Reproducing the Blondin et al. model allows us making close connection to supernova conditions. The experimental setting is then obtained by removing gravity. In this case, we impose a time-dependent supersonic inflow at the outer boundary perturbed around nominal values. At the inner boundary, we set a fixed velocity and pressure and zero density gradient.

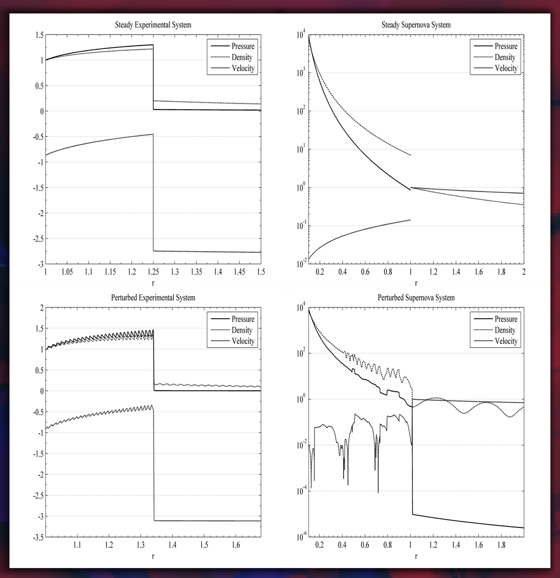

Fig. 3: Typical solution profiles for perturbed and unperturbed flows in experimental and supernova settings.

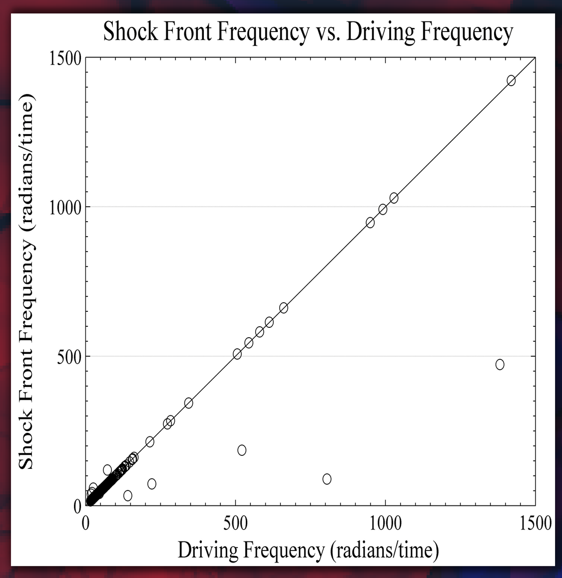

Fig. 4: Relationship between shock front oscillation frequency and upsteam density perturbation frequency in one dimension.

For both settings the initial conditions were generated by numerically solving the system of ODEs in radius obtained from the steady state version of the Euler equations. Integrating this system was done using the Dormand-Prince RK45 scheme starting from the shock and integrating toward each boundary, with the initial conditions set to either upstream (outward integration) or downstream (inward integration) from the shock. Both solutions are related through the Rankine–Hugoniot relations for a steady shock.

In the experimental parameter study, we varied the flow variables in the upstream from the shock, the ideal gas gamma (ratio of specific heats), and the aspect ratio of the domain. Upstream density perturbations were generated as a single sine wave and scanned over both amplitude and frequency. The above parameter space was probed using Latin Hypercube Sampling with the uniform probability distributions of parameters.

In 2D, the experimental setup utilizes the same boundary conditions at the inner and outer boundary with zero tangential velocity. We impose reflecting boundary conditions in angle. Parameters for the 2D runs were chosen from select 1D runs that were deemed optimal for the current application.

In multidimensional settings, non-radial hydrodynamic instabilities may appear. In the AAC theory, convective instabilities serve as the primary mechanism for energy transport in the post-shock region. The characteristic "rumbling" of the shock is due to the motion of these convective cells and interaction between the shock and sound waves.

Formation of non-radial instabilities are of the utmost importance in the experimental setting. Therefore, our aim is to verify that the conditions are conducive to the development of these instabilities. In order to investigate stability of the shock, we perturb the flow by introducing "egg crate" density perturbations upstream of the shock. The velocity and pressures are held constant in this regime. We deem this reasonable because in the experimental setting the inflowing plasma would have been overrun by the initial colliding shocks.

$$\frac{\partial E}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(F_K + F_P)\right] - E_p = 0$$
$$\frac{\partial}{\partial t}\left[\overline{\rho c_v T} + \frac{1}{2}\overline{\rho(u^2+w^2)}\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(F_C + F_K)\right] = 0$$
$$F_C = -c_p\overline{\rho w T'} \qquad F_K = -\frac{1}{2}\overline{w(\rho u_i u_i)}$$
$$F_P = \overline{w P'} \qquad E_P = \overline{P'\left(\frac{\partial u_i}{\partial x_i}\right)}$$

Eqn. 2: Kinetic energy and total energy equations with decomposed fluxes. Overbars denote averages in lateral direction. Primes denote deviation from the lateral averages.

In order to analyze the 2D simulations, we follow Hurlbert et al.'s [6] analysis of a stratified, three layer system in which a convectively unstable layer is sandwiched between two stable layers. This type of behavior is expected in the supernova; a warm, convective layer existing between the cold, accreting surroundings and the neutrino cooled region at the bottom. In the experimental setting, the large scale pressure gradient in the post-shock region may act similar to gravity by allowing buoyancy effects in the inhomogeneous density field.

Hurlbert et al.'s methodology consists of first averaging the Euler equations over the lateral direction. Then, the energy equation is rewritten as a combination of different flux gradients and energy source terms. This view allows us to easily differentiate differing flow regimes (momentum dominated, pressure dominated, etc.).

In particular, one should expect a flow field to emerge that is dominated by kinetic flux (F_k)prior to the shock and near the inner boundary, but with convective flux dominating in the middle layer. dominated by convective flux (F_c)in between.

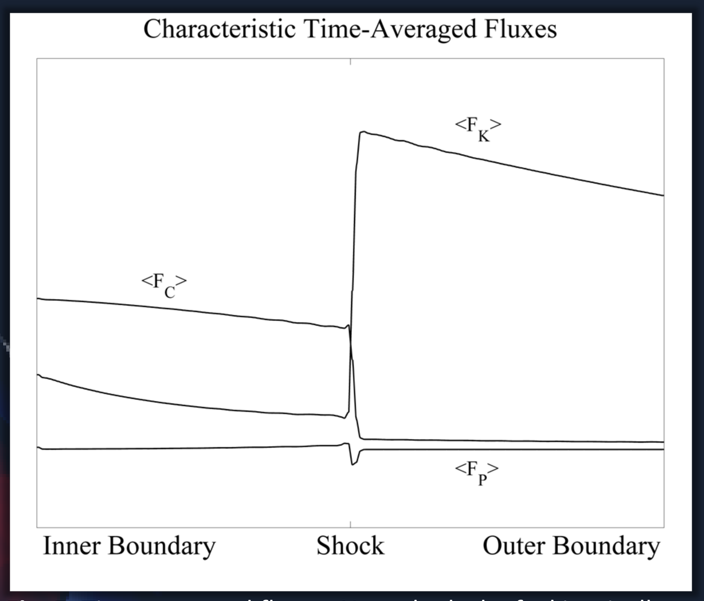

Fig. 5: Time-averaged fluxes. Note the lack of a kinetically dominated region to the left of the shock.

RESULTS We found that in the experimental system the shock tends to drift with time, and eventually leaves the domain. Due to this behavior, we had to construct a suitable metric for characterizing shock stability. For the present application, we use the instantaneous shock radius. The shock was considered steady while within ±5% of its initial radius. Before the frequency of the shock front could be determined, the drift over time needs to be removed. This was accomplished using a Butterworth high-pass filter with a cut-off frequency of 30, which is a heuristic upper bound for the shock drift. Due to sampling the shock position at unequal time intervals, the Lomb-Scargle method of least-squares spectral analysis was used to construct the power spectrum. This is also enables hypothesis testing by determining the likelihood of the if same results given a random distribution.

In order for post-shock flow structure to be indicative of larger structure formation, it should decouple from the upstream perturbations. It is conceivable that those new structures could perturb the shock at frequencies different from the driving frequency. However, our simulations results (Fig. 4) indicate that in most cases (see below discussion of exceptions) the shock oscillates with the frequency equal to the driving frequency. We conclude that the shock is stable to radial perturbations and evolves with no noticeable feedback from the perturbed post-shock region.

The data points that are not consistent with 1:1 coupling of the shock oscillations to the upstream perturbations have hypothesis test values multiple orders of magnitude greater than the other results. This implies that they are most likely numerical artifacts. We have confirmed this conclusion through the visual inspection of the corresponding hydrodynamic models by failing to find evidence for perturbation growth in the post-shock region. From these results, it is reasonable to assume that the outlying models are not indicative of the true behavior of these model experimental systems.

The time at which the shock leaves determines the maximum time available for studying its stability, τ_{max}. From this the number of advection times may be determined as $N_{adv} = \tau_{max}/\tau_{adv}$. Using this measure, we obtain $N_{adv} \sim O(10)$ for typical system parameters. We conclude that systems that admit many advective crossing times are not only possible to obtain, but can be easily found in the parameter space.

From the flux analysis, we note that no unstable convective layer emerges. While there is an increase in the convective flux behind the shock, the third layer of kinetically dominated flow does not manifest. In order to explain this, we consider the entropy profiles of each setting. In the supernova setting, a large scale negative entropy gradient exists in the post-shock region. This is due to redistribution of the fluid flow energy components. This behavior could be understood as follows. Consider adding air to a hot air balloon while standing on the surface of the earth. The local bubble of higher entropy gas (the balloon) has a preferred state at higher altitudes, which manifests itself as buoyant forces that cause it to rise. In the experimental system, there is a positive entropy gradient in the post-shock region. This infers that the preferred state of shocked fluid parcels is to continue falling towards the inner boundary. Therefore, we conclude that it is not possible to generate a convectively unstable flow configuration in the experiment as described here. We are currently considering modifications to this experimental design that would bring the system closer to the supernova setting.

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IMAGES

(Center) 3D supernova model courtesy of T. Plewa
(Bottom) Flow variables for 2D experimental setting. Clockwise: density, radial velocity, entropy, pressure