Investigating the Effects of Inhomogenous Porosity on Resin Infusion

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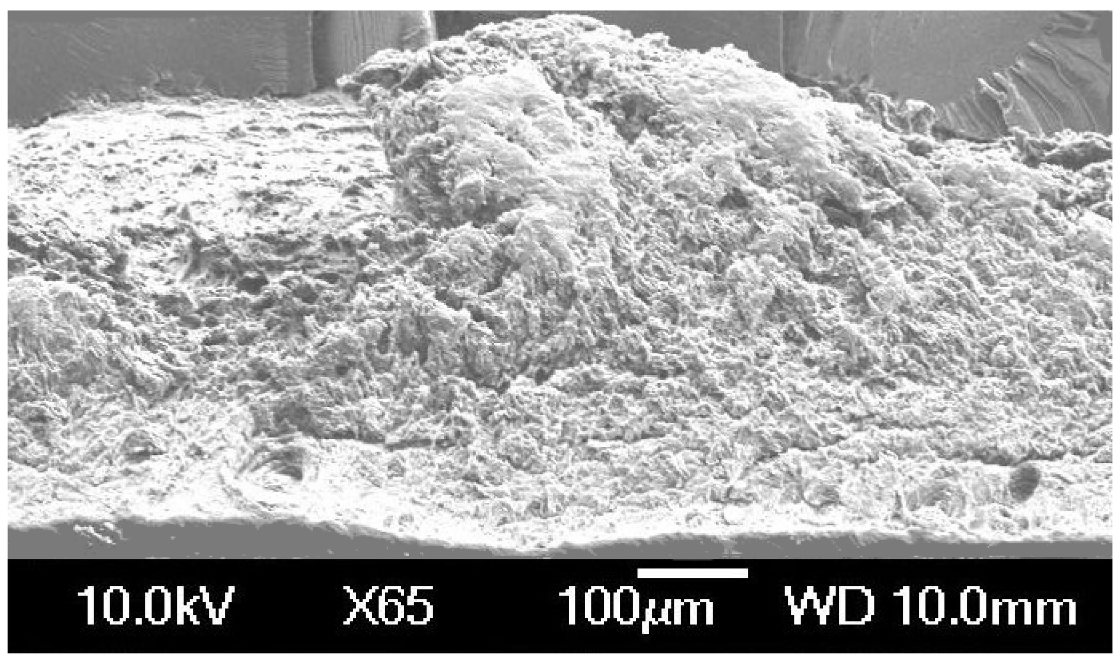
Abstract

During a resin infusing process a polystyrene foam is located between an immovable flat lower surface and an initially flat vacuum bag on top. After infusion the top surface becomes distorted. It is expected that variations in the porosity cause the foam to expand or contract unevenly during the infusion process. As resin is injected into the foam it will flow according to the Darcy-Brinkmann equation for porous media. The resulting pressure field can expand or contract the pores, thus changing the overall shape of the foam. We wish to validate this theory by using the Deal.ii finite element library to approximate the solution of a system of partial differential equations over a sequence of computational domains.

Introduction

Foam infused with resin is viewed as a potential way to control placement of additives inside a larger structure. For example if one wanted to mechanically reinforce one particular area of an airplane fuselage for instance, one could place a foam containing carbon nanotubes at that point before resin infusion. In many applications, such as the one mentioned, it is important that the surface of the hardened resin remains smooth. It has been experimentally shown that in the area above the foam this is not the case. We wish to explain why this might be and create a computational model to validate our hypothesis.



Model

Hypothesis

It is believed that nonuniform porosity could be a cause of surface roughness. As the fluid flows through the foam it creates a pressure profile which expands and contracts the pores. Since the bottom surface is on an immovable flat surface, this expansion or contraction manifests itself as a roughening of the top surface.

Fluid Motion

The velocity and pressure fields for steady flow through porous media are found by solving the Darcy-Brinkman equations:

$$\begin{aligned} &\frac{\mu}{\kappa(\mathbf{x})}\mathbf{u} - \mu\Delta\mathbf{u} + \nabla p = \mathbf{0} \\ &\nabla \cdot \mathbf{u} = 0 \end{aligned} \qquad (1)$$

where

- $\mathbf{u} = \langle u, v\rangle$ is the velocity of the fluid (in 2 dimensions)
- p is the pressure
- μ is the dynamic viscosity of the fluid
- $\kappa(\mathbf{x})$ is the permeability of the media, which we take to be a scalar that varies in space

Porosity

Next, let $\phi(\mathbf{x})$ be the porosity of the foam, which varies in space. Then let us assume that the permeability $\kappa(\mathbf{x})$ is related to the porosity by:

$$\kappa(\mathbf{x}) = \alpha\phi(\mathbf{x})^{\beta} \qquad (2)$$

where α and β are constants, which we will prescribe later.

We also need to define an initial porosity field, $\phi_0(\mathbf{x})$. Given N random generator points $\mathbf{x}_i$ $(0 \le i \le N)$ in our domain, generate a circular pore at each $\mathbf{x}_i$ and then give them a random inner radius r_i^{in} and a random outer radius r_i^{out}. For each pore, inside its inner radius, the porosity is 1 (full void), and outside its outer radius the porosity is 0. Between its inner and outer radii the porosity decreases linearly from 1 to 0. Mathematically this can be described as:

$$\phi_0(\mathbf{x}) = \min\left(\max\left(\sum_{i=0}^{N}\sigma_i(\mathbf{x}), \phi_{\text{min}}\right), 1\right)$$

where:

$$\sigma_i(\mathbf{x}) = \begin{cases} 1 & \text{if } |\mathbf{x}-\mathbf{x}_i| \le r_i^{\text{in}} \\ \frac{|\mathbf{x}-\mathbf{x}_i| - r_i^{\text{out}}}{r_i^{\text{in}} - r_i^{\text{out}}} & \text{if } r_i^{\text{in}} \le |\mathbf{x}-\mathbf{x}_i| \le r_i^{\text{out}} \\ 0 & \text{otherwise} \end{cases}$$

Domain Height

After infusion, the pressure of the resin expands the pores according to the following equation:

$$\phi(\mathbf{x}) = \phi(\mathbf{x})_0 \exp\left(\frac{p(\mathbf{x}) - p_0}{G}\right) \qquad (3)$$

where $p(\mathbf{x})$ is the local pressure, p_0 is the initial pressure, 1 atm, before the resin infusion and G is the foam modulus.

Using this new porosity, we can update the height using the integral:

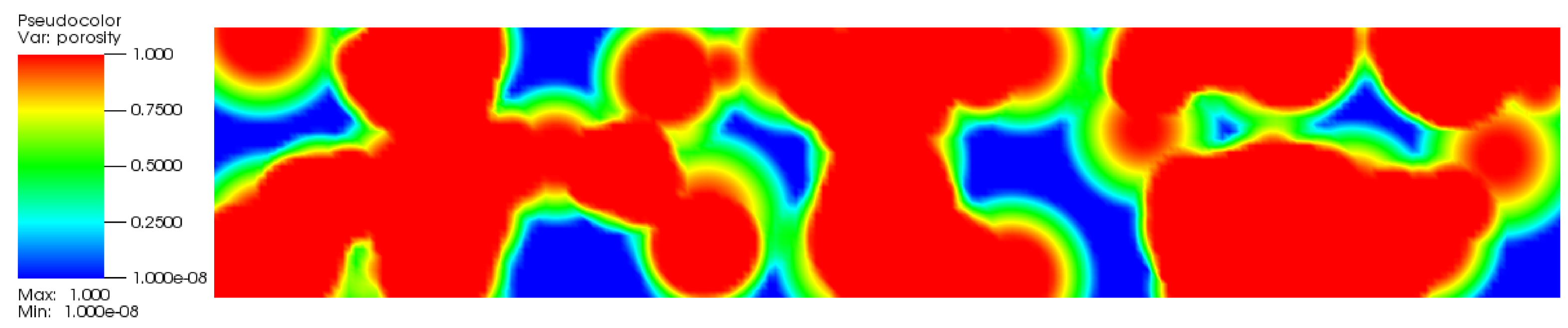
$$\frac{H_{k+1}(x)}{H_0} = \int_0^{H_k(x)} \frac{\phi(\mathbf{x})}{\phi_0(\mathbf{x})} dy \qquad (4)$$

where H_0 is the initial height, and $H_k(x)$ is the foam height for the k^{th} iteration. Note that $\phi_0(\mathbf{x})$ is the initial porosity mapped to the domain given by $H_k(x)$. We solve (1) on each new domain and keep updating the domain until the difference between H_{k+1} and H_k reaches a prescribed tolerance.

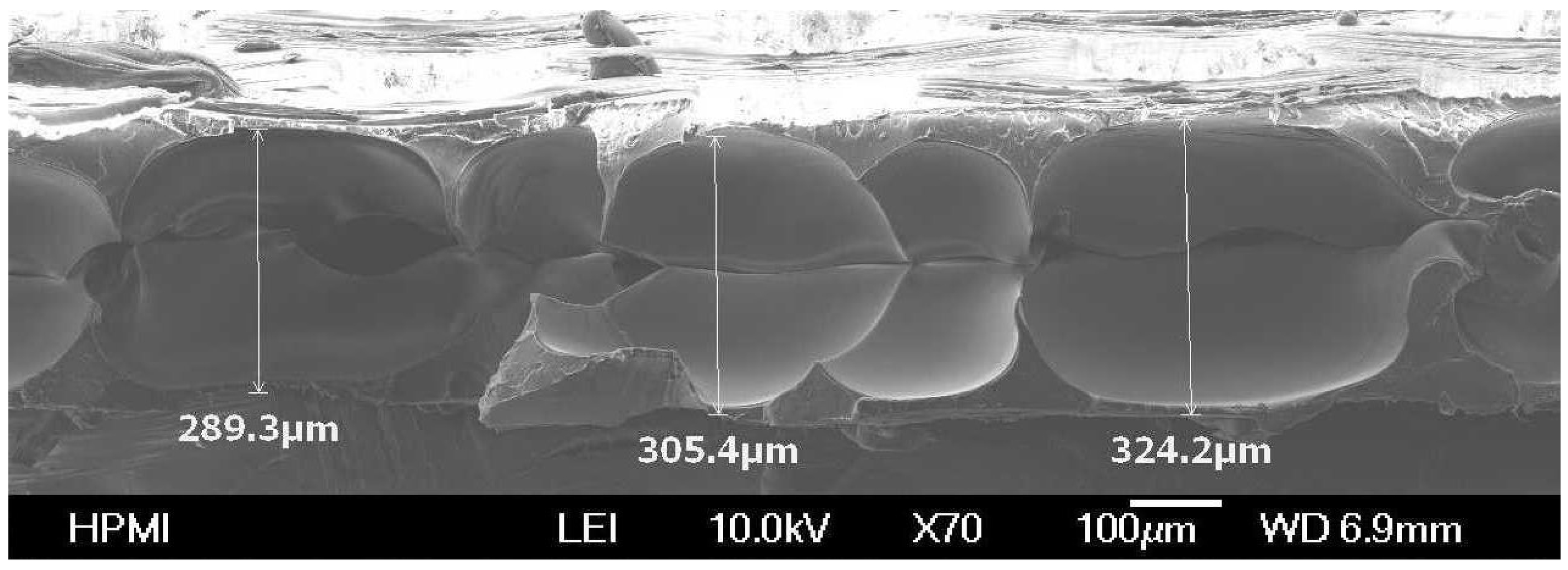
Results

Using the parameters listed below we can generate the following initial porosity field:

initial foam height	1 mm
foam length	5 mm
α	2×10^{-4}
β	2
N	50
r_{in}	[0.001 mm, 0.03 mm]
r_{out}	[0.05 mm, 0.12 mm]



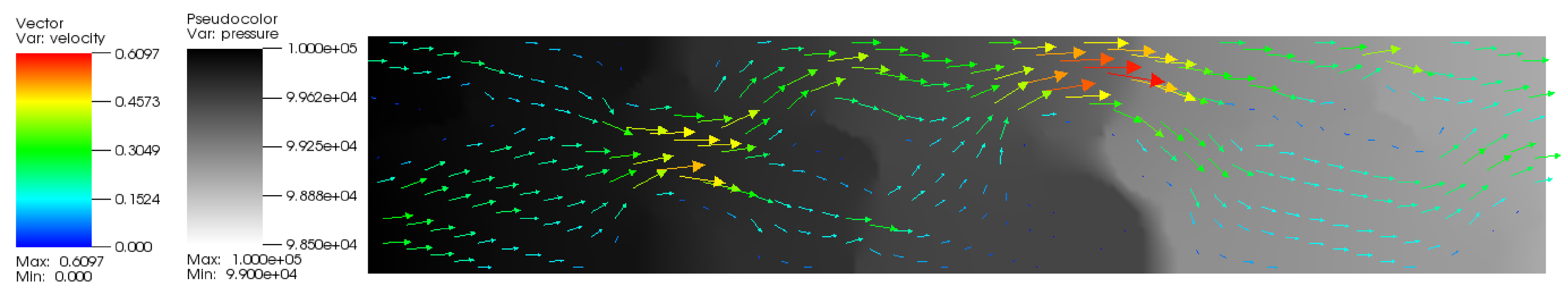
This field has an average porosity of 0.7729 and an average permeability from (2) of 1.432×10^{-4} mm^2. It can be compared to an SEM image of a foam that was held under a low temperature mold:



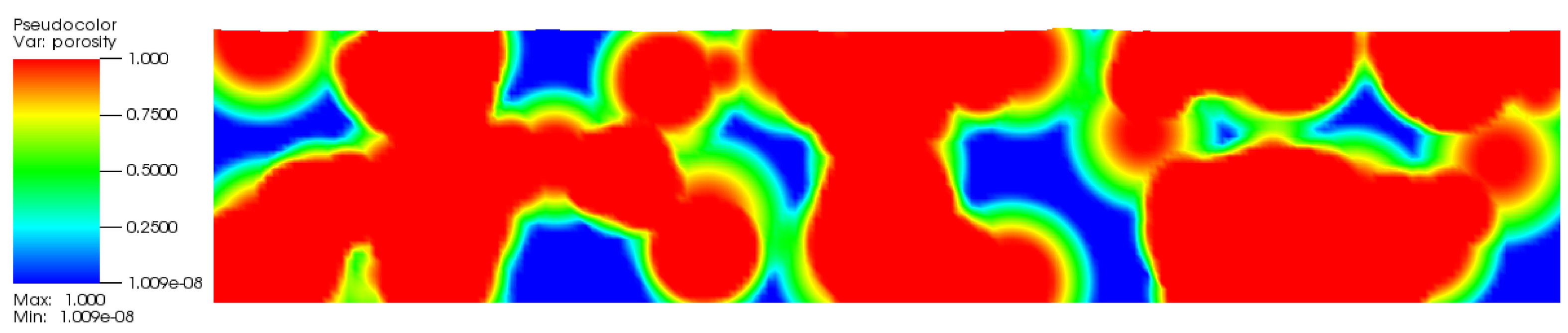
Solving (1) using finite elements with this porosity field and the following material properties and boundary conditions gives pressure and velocity profiles:

viscosity	0.1 Pa s
modulus	1×10^5 Pa
inlet normal stress	1×10^5 Pa
outlet normal stress	9.9×10^4 Pa

Due to the finite element formulation, in order to have physical boundary conditions at the inlet and outlet we have to prescribe the normal stresses there. As we will see this is almost equivalent to setting the pressure, particularly if the change in normal stress is not too large. In addition we prescribe no-slip boundary conditions on the top and bottom of the foam and no vertical velocity at the inlet and outlet.

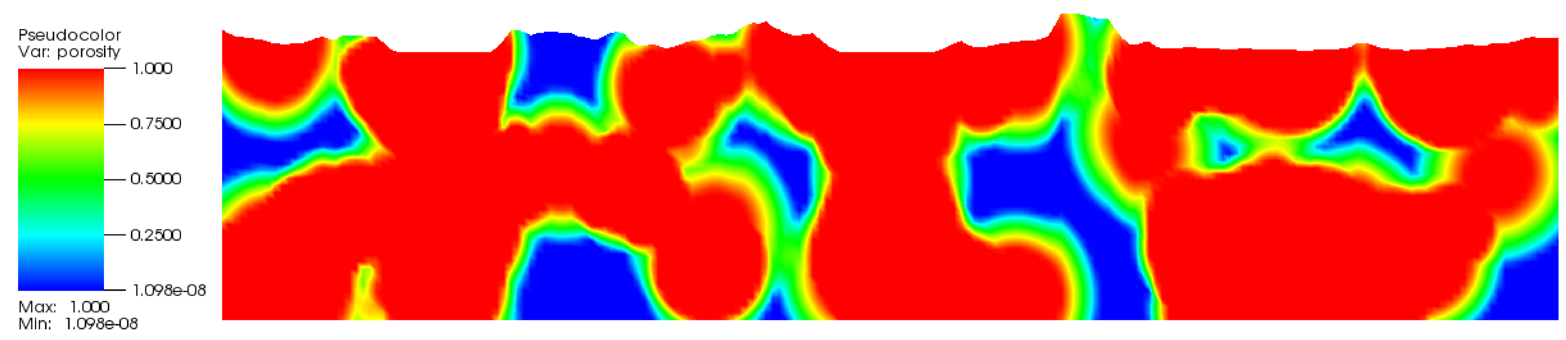


Next we use (3) to update the porosity field, and (4) to update the domain height.



The change in domain shape is small, 3.964×10^{-4} mm on average, but it is there. Resolving (1) on this new domain and updating the domain a second time gives no appreciable change in domain shape, so we can conclude the scheme converges after 1 iteration.

If we wish to see a larger change in domain shape, we can decrease the foam modulus G. With $G = 1 \times 10^4$ Pa, we get the following final domain shape and porosity field, again after 1 iteration:



Conclusions

Using the model described above, we see that the inhomogenous porosity does have an effect on the shape of the foam. With the original estimate of the foam stiffness however, this fails to account for the size of the ridges, which in experiments can be up to 20% of the domain height.

Future Research

This simple model fails to account for the viscous stresses of the resin on the foam. Future work will look at these effects, and relate this stress to strain on the foam in order to compute the deformation.