Abstract

We apply a high-order boundary integral equation method to simulate multicomponent, semipermeable vesicles in various Stokes flows. Our multicomponent vesicles are representative of cell membranes containing aquaporins and are only permeable to water. This semipermeability is important for many biophysical processes including cell migration and cell rupture. Our semipermeability model depends on the membrane forces where the fluid flux is proportional to the pressure drop. The multicomponent model uses the Cahn-Hilliard equation to allow for phase separation between different lipid species. We consider vesicles in quiescent flow and shear flow.

Introduction

Vesicles are deformable capsules that are:

- Submerged in and filled with an incompressible viscous fluid
- Resist bending
- Locally inextensible
- Used to model biomembranes such as red blood cells

Governing Equations

Following the work of Sohn et al. JCP, 2010, we define the lipid energy as

\[ E^f = \frac{1}{2} \int f(u) \frac{1}{\gamma^2} (1 - \gamma^2)^2 \, ds, \]

where \( f(u) \) represents the free energy, and \( \gamma \) is the double well potential.

The fluid and vesicle equations are

\[ \rho \dot{u} - \nabla p = \mu \nabla^2 u, \quad \nabla \cdot u = 0, \quad f_s = \gamma, \quad \nabla \cdot f_s = 0, \quad \gamma \in \Omega, \quad \text{conservation of momentum,} \]

\[ \rho \dot{\gamma} = \frac{\gamma}{\gamma^2} \left( \int (f(u) - f_s) \, ds \right) - \gamma \nabla \cdot f_s, \quad \gamma \in \Gamma, \quad \text{dynamical equation for the vesicle,} \]

where \( \rho \) is the fluid density, \( \mu \) is the fluid viscosity, \( \gamma \) is the bending modulus, and \( f_s \) is the bending energy.

\[ \text{and have the conditions} \]

\[ \nabla \cdot \dot{u} = 0, \quad \text{velocity continuity,} \]

\[ \dot{\gamma} \rightarrow \infty \quad \text{far-field condition.} \]

We do a change of variables to the \( \theta - \Lambda \) formulation. After this change of variables, the dynamics of \( \theta \) and \( \Lambda \) are governed by

\[ \frac{\partial \theta}{\partial t} = \left( V + \frac{\partial}{\partial \Lambda} \theta \right), \quad \theta \in \Omega, \quad \text{conservation of mass,} \]

\[ \frac{\partial \Lambda}{\partial t} = \left( T_e + \kappa (V + \frac{\partial}{\partial \Lambda} \Lambda) \right), \quad \Lambda \in \Gamma, \quad \text{velocity continuity,} \]

with the inextensibility condition

\[ T_e + \kappa (V + \frac{\partial}{\partial \Lambda} \Lambda) = 0. \]

To reduce the stiffness of the system, we extract the dominant terms in the governing equations at small spatial scales.

\[ V \approx \frac{n_{max}}{n_{min}} \left( \frac{\partial \Lambda}{\partial \Lambda} \right), \quad \text{where} \]

\[ C^f(a) = \int_{\Omega} \left( f(u) - f_s \right) \, ds. \]

The reduced area of the vesicle is defined as \( \Delta A \), where \( \Delta A \) is the area of the vesicle and \( A_0 \) is the area of a circle with the same length as \( \gamma \).

Discussion

- Have developed integral equation methods to simulate multicomponent semipermeable vesicles.
- In a quiescent flow, single-component vesicles inflate to a circle, but multicomponent vesicles inflate to non-circular shapes.
- In a shear flow, semipermeable vesicles tank tread or tumble depending on the lipid concentration.